Electron and ion thermal forces in complex (dusty) plasmas

Sergey A. Khrapak*
Max-Planck-Institut für extraterrestrische Physik, D-85741 Garching, Germany
(Dated: September 19, 2012)

Expressions for the ion and electron thermal forces acting on a charged grain, suspended in a weakly ionized plasma subject to temperature gradients, are derived. These forces can significantly affect grain transport in many realistic situations, ranging from complex plasmas in laboratory and microgravity experiments, to charged dust in planetary atmospheres and fusion devices.

PACS numbers: 52.27.Lw, 52.25.Vy, 94.05.Bf

The momentum exchange between different species plays an exceptionally important role in complex (dusty) plasmas – multi-component systems consisting of charged micron-sized ("dust") grains embedded in a plasma [1–3]. In a weakly ionized plasma, the momentum transfer in collisions with the neutral gas "cool down" the system, in particular grains and ions, and introduce some damping. In addition, ion and electron flows are usually present in plasmas due to global electrical fields (e.g. ambipolar or sheath fields in gas discharges). The momentum transfer from flowing ions and electrons can become a dominant factor affecting and regulating static and dynamical properties of the grain component. A remarkable example wherein the momentum exchange in ion-grain collisions plays a crucial role is the formation of a void in the central part of an rf discharge in experiments under microgravity conditions [4–7]. In this case, the momentum transfer rate from the ions streaming outwards exceeds the electrical force (pointing to the center), leaving a void.

Normally, the momentum transfer is associated with the relative motion between the grain and other plasma components. The related forces are then called "drag forces" (terms friction and resistance force are also relevant, especially when the grain moves with respect to the stationary plasma background). Usually the mean free paths of all plasma species (neutrals, ions, and electrons) are very long compared to the grain size [$\sim \mathcal{O}(\mu\mathrm{m})$], which corresponds to the limit of large Knudsen number. The neutral drag force in this regime was calculated almost a century ago by Epstein [8] and is often referred to as Epstein drag. The ion drag force is considerably more complicated issue. First, in addition to direct ion-grain collisions, elastic ion scattering in the electrical potential around the grain has to be taken into account. Second, the ion-grain interaction under typical experimental conditions is not weak, which implies that the conventional theory of Coulomb scattering is inapplicable to describe elastic ion-grain collisions [9, 10]. This explains why the problem of the accurate determination of the ion drag force in complex plasmas has received considerable attention in the last decade, including various theoretical approaches [9–14], numerical simulations [15–17] and experiments [18–23]. Moreover, theoretical results on the ion-grain scattering in the regime of strong (Yukawa) interaction found their application not only in the field of complex plasmas, but also in the context of quark-gluon plasma [24], scattering of dark matter particles [25], and conventional strongly coupled plasmas [26]. Regarding the momentum transfer in electron-grain collisions, it often plays only a minor role due to the small electron mass, although this is not always the case [27].

The relative motion between the grain and other plasma species is not the only mechanism which can be responsible for the momentum transfer. The latter can occur even in the absence of net flows of plasma species relative to the grain. A relevant example is the thermophoretic force (terms "thermal force" and "radiometer force" are also employed in the literature [28]). It describes the phenomenon wherein small particles, suspended in a gas where a temperature gradient exists (but macroscopic flows are absent), experience a force in the direction opposite to that of the gradient. Elementary consideration of this phenomenon has been already given by Einstein [29] and then many others investigated this topic in detail (see e.g. Refs. [30–32] and references therein). In the context of complex plasmas, Jellum *et al.* [33] were apparently the first who recognized the possibility to manipulate the particles in gas discharges using the thermophoretic force. Since then, applying the vertical temperature gradients to compensate for the particle gravity has become a standard technique for controlled particle manipulation in laboratory experiments [34–37].

In complex plasmas, the thermophoretic force has its counterparts associated with the charged electron and ion components, provided the corresponding temperature gradients are present. We will refer to these forces as the electron and ion *thermal* forces to distinguish them from the conventional drag forces. To the best of our knowledge, these forces have not yet been considered in the literature in any reasonable detail. The main purpose of this Letter is to fill this gap. In the following we derive the expressions for the ion and electron thermal forces acting on a highly charged grain immersed in a weakly ionized plasma subject to temperature gradients. We analyze the directions and the magnitudes of these forces and show that they exhibit interesting and non-trivial behavior. Comparison with other forces indicates that

*Also at Joint Institute for High Temperatures, Moscow, Russia

the thermal forces can provide important contributions to the net force balance in complex plasmas.

The general expression for the force associated with the momentum transfer from a light species α to the massive grain at rest is

$$\mathbf{F}_\alpha = m_\alpha \int \mathbf{v} v \sigma_\alpha(v) f_\alpha(\mathbf{v}) d^3 v, \quad (1)$$

where m_α, $f_\alpha(\mathbf{v})$ and $\sigma_\alpha(v)$ are the corresponding mass, velocity distribution function, and (velocity dependent) momentum transfer cross section ($\alpha = n, i, e$ for neutrals, ions, and electrons, respectively). The net momentum transfer occurs when the velocity distribution has some asymmetry (e.g. relative motion). Assuming weak asymmetry, we write $f_\alpha(\mathbf{v}) \simeq f_{\alpha 0}(v) + f_{\alpha 1}(\mathbf{v})$, where the symmetric component $f_{\alpha 0}$ is taken to be Maxwellian

$$f_{\alpha 0} = n_\alpha (m_\alpha / 2\pi T_\alpha)^{3/2} \exp(-m_\alpha v^2 / 2T_\alpha).$$

Here n_α and T_α are the density and temperature (in energy units) of the species α. The asymmetric component $f_{\alpha 1}$, which gives contribution to the integral in (1), depends on the nature of the anisotropy. In the case of subthermal drifts with relative velocity $\mathbf{u}_\alpha$ ($u_\alpha \lesssim v_{T_\alpha}$) it reduces to $f_{\alpha 1} \simeq f_{\alpha 0}(\mathbf{v}\mathbf{u}_\alpha / v_{T_\alpha}^2)$, where $v_{T_\alpha} = \sqrt{T_\alpha / m_\alpha}$ is the thermal velocity. This ansatz corresponds to the conventional calculation of the neutral, ion and electron drag forces and has been thoroughly investigated earlier [2, 3, 8, 9, 11, 27]. If the drift is not subthermal, nonlinearized shifted Maxwellian distribution function [14] or another distribution function, appropriate for the situation considered [13, 38, 39] has to be used in Eq. (1).

The focus of this Letter is on a complementary situation when relative drifts are absent, but the momentum transfer do occur due to the net momentum flux caused by the temperature gradients. In this case, the asymmetric part of the velocity distribution function of the component α can be approximated as

$$f_{\alpha 1} \simeq \frac{m_\alpha \kappa_\alpha f_{\alpha 0}}{n_\alpha T_\alpha^2} \left[1 - \frac{m_\alpha v^2}{5 T_\alpha} \right] \mathbf{v} \nabla T_\alpha, \quad (2)$$

where κ_α is the thermal conductivity of the species α. This form ensures that the self-consistent density gradient and/or electric field (for charged species), build up in response to the temperature gradient, result in no net flux: $\mathbf{j}_\alpha = \int \mathbf{v} f_{\alpha 1} d^3 v = 0$ (i.e., $\mathbf{u}_\alpha = 0$). In addition, Eq. (2) obviously satisfies the Fourier's law for heat transfer: $\mathbf{q}_\alpha = \int (m_\alpha v^2/2) \mathbf{v} f_{\alpha 1} d^3 v = -\kappa_\alpha \nabla T_\alpha$. Equation (2) is an approximation, which is exact only for the special case of $\propto r^{-4}$ interactions (for the rigorous mathematical treatment of non-uniform gases see Ref. [40]). Its accuracy is, however, more than acceptable for our present purposes, especially in view of further simplifications involved in treating electron- and ion-grain collisions.

Substituting Eq. (2) into (1) we get after integration over the angles in spherical coordinates

$$F_{\mathrm{T}\alpha} = \frac{16 \kappa_\alpha \nabla T_\alpha}{15 \sqrt{2\pi} v_{T_\alpha}} \int_0^\infty x^2 (\tfrac{5}{2} - x) \exp(-x) \sigma_\alpha(x) dx, \quad (3)$$

where the remaining integration is over the reduced velocity $x = v^2 / 2 v_{T_\alpha}^2$. Let us first apply this ansatz to calculate the thermal force acting on a non-charged particle in a non-ionized atomic gas – thermophoretic force. In this case the momentum transfer cross section is velocity-independent, $\sigma_n(x) = \pi a^2$, and the integration yields $F_{\mathrm{T}n} = -(8\sqrt{2\pi}/15)(\kappa_n a^2 / v_{T_n}) \nabla T_n$, where a is the particle radius. This coincides with the celebrated expression by Waldmann [30]. The force pushes the particles into the region with lower gas temperature. Physically, this is because hotter atoms transfer more momentum to the grain than the colder ones.

It is convenient to write the generic expression for the thermal forces in complex plasmas as

$$F_{\mathrm{T}\alpha} = -\frac{8\sqrt{2\pi}}{15} \frac{\kappa_\alpha a^2 \nabla T_\alpha}{v_{T_\alpha}} \Phi_\alpha. \quad (4)$$

For the neutral component (thermophoresis) $\Phi_n = 1$. The factors $\Phi_{i(e)}$ account for the electrical interactions between the ions (electrons) and the charged grain. They depend on the shape of the plasma electrical potential around the grain. We use the conventional Debye-Hückel (Yukawa) form, $\phi(r) \simeq (Q/r) \exp(-r/\lambda)$, where Q is the grasin charge and λ is the plasma screening length. In low-temperature gas discharges the dominant charging process is the continuous absorption of electrons and ions on the grain surface. In this case the charge is negative and proportional to the product of the grain radius and the electron temperature, viz. $Q = -z(aT_e/e)$, where z is the coefficient of order unity, which depends on plasma parameter regime [41]. The screening comes from redistribution of plasma electrons and ions in the vicinity of the charged grain. In the absence of substantial ion flows and strong nonlinearities in ion-grain interaction [41–43], the screening length is approximately given by the ion Debye radius $\lambda \simeq \lambda_{\mathrm{D}_i} = \sqrt{T_i / 4\pi e^2 n_i}$, provided $T_e \gg T_i$. Although it is well recognized that the long range asymptote of the electrical potential can be modified (by e.g. continuous plasma absorption on the grain surface [44–47] or plasma ionization/recombination effects [48]), this is expected to affect merely grain-grain interactions, but not the momentum transfer from the ions and electrons.

Electrons. Normally, the grain radius in complex plasmas is much smaller than the plasma screening length. In this case, the electron-grain interaction can be called "weak", in the sense that its range – the Coulomb radius $R_{\mathrm{C}}^e \sim za$ – is much smaller than the screening length λ [3]. This implies that the Coulomb scattering theory is appropriate to describe electron-grain elastic collisions. The general momentum transfer cross section for scattering in the Coulomb potential Q/r is

$$\sigma_\alpha^{\mathrm{s}}(v) = 4\pi R_\alpha^2(v) \ln \left[\frac{R_\alpha^2(v) + \rho_{\max}^2(v)}{R_\alpha^2(v) + \rho_{\min}^2(v)} \right]^{1/2}, \quad (5)$$

where $R_\alpha(v) = (|Q|e / m_\alpha v^2)$ and $\alpha = e, i$. The maximum impact parameter $\rho_{\max}$ is necessary to avoid the

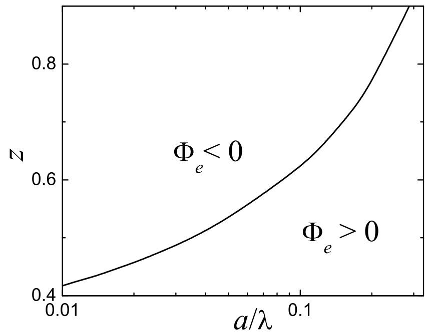

FIG. 1: The transition curve separating regions of positive and negative electron thermal force in the plane of reduced grain size a/λ and charge z.

logarithmic divergence of the cross section. In the standard Coulomb scattering theory $\rho_{\max} = \lambda$, and this choice is appropriate for the weak electron-grain interaction [3, 27]. The minimum impact parameter $\rho_{\min}$ is zero in the standard Coulomb scattering theory. In the considered case, however, some electrons are falling onto the grain instead of being elastically scattered in its electrical potential. Consequently, $\rho_{\min}$ should be set equal to the maximum impact parameter corresponding to electron collection by the grasin, $\rho_{\rm c}$. The orbital motion limited (OML) theory yields [49] $\rho_{\rm c}(v) = a\sqrt{1-2R_e(v)/a}$ for $2R_e(v) < a$ and $\rho_{\rm c} = 0$ otherwise (sufficiently slow electrons cannot be collected even in head-on collisions with the grain due to electrical repulsion). The momentum transfer cross section for electron collection is simply $\sigma_e^{\rm c}(v) = \pi\rho_{\rm c}^2(v)$. Expressing now velocity in terms of $x = v^2/2v_{T_e}^2$, substituting $\sigma_e(x) = \sigma_e^{\rm s}(x) + \sigma_e^{\rm c}(x)$ into Eq. (3) and performing the integration with the appropriate limits yields:

$$\Phi_e = (1 + \tfrac{3}{2}z + z^2)\exp(-z) - \tfrac{z^2}{2}\Lambda_e, \tag{6}$$

where Λ_e is the electron-grain Coulomb logarithm

$$\Lambda_e = \int_0^\infty h(x)\ln\left(1 + \tfrac{4\lambda^2}{a^2}\tfrac{x^2}{z^2}\right)dx - 2\int_z^\infty h(x)\ln\left(\tfrac{2x}{z} - 1\right)dx. \tag{7}$$

The function $h(x)$ is defined as $h(x) = \left(\frac{5}{2} - x\right)\exp(-x)$. Equations (4), (6) and (7) constitute the expression for the electron thermal force.

For an uncharged particle ($z = 0$) we have $\Phi_e = 1$, as expected. In this case the electron thermal force pushes the grains in the direction of lower electron temperature, similarly to the thermophoretic force. According to Eq. (6) the contributions from collection and scattering are directed oppositely to each other. For a sufficiently high charge, the scattering part becomes dominant and the force reverses direction. The physical reason is that the (Coulomb) scattering momentum transfer cross section quickly decreases with velocity, so that the cold electrons are more effective in transferring their momentum upon scattering. In this regime ($\Phi_e < 0$), the thermal force acts in the direction of higher electron temperature. In Figure 1 the curve separating the positive and negative values of Φ_e is plotted in the plane (a/λ, z). For most experimental conditions we expect $\Phi_e < 0$, although the transition line does not seem unreachable [50].

Ions. The Coulomb radius of ion-grain interaction $R_{\rm C}^i \sim za\tau$ is not necessary small compared to the plasma screening length due to the presence of a (normally) large factor $\tau = T_e/T_i$ – electron-to-ion temperature ratio [3]. The interaction range can exceed λ and considerable amount of momentum transfer can occur for impact parameters beyond λ. This implies that the standard Coulomb scattering approach is inappropriate [9, 10]. It makes sense to consider two regimes of ion scattering separately. In the regime of *moderate ion-grain interaction*, an extension of the standard Coulomb scattering theory is possible by taking into account the momentum transfer from the ions that *approach* the grain closer than λ [9]. This results in $\rho_{\max} = \lambda\sqrt{1+2R_i(v)/\lambda}$. This approximation demonstrates good accuracy for $R_i(v) \lesssim 5\lambda$ and reduces to $\rho_{\max} = \lambda$ in the limit of weak ion-grain interaction [$R_i(v) \ll \lambda$]. The impact parameter corresponding to the ion collection is $\rho_{\rm c}(v) = a\sqrt{1+2R_i(v)/a}$ and the corresponding collection cross section is $\sigma_{\rm c}(v) = \pi\rho_{\rm c}^2(v)$. Combining the contributions from collection and scattering yields:

$$\Phi_i = 1 - \tfrac{1}{2}z\tau - z^2\tau^2\Lambda_i, \tag{8}$$

where Λ_i is the (modified) ion-grain Coulomb logarithm

$$\Lambda_i = \int_0^\infty h(x)\ln\left[\frac{2x(\lambda/a) + z\tau}{2x + z\tau}\right]dx. \tag{9}$$

In typical complex plasmas with $\lambda \gg a$, $\tau \gg 1$, and $z \sim 1$, the Coulomb logarithm can be roughly estimated as $\Lambda_i \simeq \ln(1+\beta_T^{-1})$, where $\beta_T = \beta(v_{T_i}) = (a/\lambda)z\tau$ and $\beta(v) = R_i(v)/\lambda$ is the ion scattering parameter [9]. The approach is reliable up to $\beta_T \lesssim 5$. Since the product $z\tau$ is normally quite large, $z\tau \sim \mathcal{O}(10^2)$, it follows that (i) the scattering provides dominant contribution to the momentum transfer; (ii) $\Phi_i < 0$, i.e. the grains are pushed into the region with higher ion temperature. The physical reason is again fast decrease of the scattering momentum transfer cross section with the ion velocity, so that cold ions transfer more momentum to the grain. This is not a unique example when the force acting on a charged grain is directed oppositely to the net ion momentum flux. Another example is related to the sign reversal of the ion drag force acting on an absorbing particle in the highly collisional (continuum) limit [41, 51–54], although the detailed physics is different.

In the regime of very *strong ion-grain interaction*, the scattering is characterized by the formation of a potential barrier for ions with impact parameters above the critical one ρ_*, which considerably exceeds λ. For $\rho < \rho_*$ scattering with large angles occurs, which gives the major contribution to the momentum transfer. Relative importance of momentum transfer from distant collisions with $\rho > \rho_*$ decreases rapidly with the increase in ion-grain interaction strength. The detailed consideration of

the momentum transfer in this regime can be found in Refs. [10, 11]. In the limit $R_i(v) \gg \lambda$ (i.e. $\beta(v) \gg 1$ [55]) the total momentum transfer cross section (collection and scattering) can be approximated as $\sigma_\Sigma(v) \simeq \pi\rho_*^2(v)$, with $\rho_*^2 \simeq \lambda^2 \left\{\ln^2[\beta(v)] + 2\ln[\beta(v)]\right\}$. The integration yields

$$\Phi_i \simeq -\left(\tfrac{\lambda}{a}\right)^2 \int_0^{x_*} x^2 h(x) \left[\ln^2\left(\tfrac{\beta_T}{2x}\right) + 2\ln\left(\tfrac{\beta_T}{2x}\right)\right] dx. \tag{10}$$

The upper limit of integration can be chosen as $x_* = \beta_T/2$ to avoid unphysical regime of negative cross section in this approximation. However, due to the presence of exponentially small term in $h(x)$, taking $x_* = \infty$ will not produce big errors for $\beta_T \gg 1$. Note that the sign of Φ_i is changed from negative to positive upon increasing β_T (in the considered approximation this happens at $\beta_T \simeq 120$; the exact value is rather sensitive to the functional dependence of the cross section on the ion velocity and, thus, is subject to significant uncertainty). Physically, the sign reversal occurs because scattering in the Yukawa potential in the limit of strong interaction tends to that on a hard sphere with the radius ρ_*, which only weakly (logarithmically) depends on the ion velocity.

Having derived the expressions for the ion and electron thermal forces, let us discuss their role in practical situations. As a first example, consider creating a gradient of the neutral gas temperature to compensate for grain gravity in low ionized laboratory gas discharges (thermophoretic levitation) [33, 34]. In this case, the ion temperature is likely coupled to the neutral gas temperature, $\nabla T_i \simeq \nabla T_n$. The neutral and ion thermal force are directed in the opposite directions (in the regime of weak and moderate ion-grain interaction). Their ratio is $|F_{\mathrm{T}i}/F_{\mathrm{T}n}| \sim (n_i/n_n)(\sigma_{nn}/\sigma_{in})z^2\tau^2\Lambda_i$, where we take into account that $\kappa_\alpha \sim (n_\alpha/n_n)(v_{T_\alpha}/\sigma_{\alpha n})$ in weakly ionized plasmas (σ_{nn} and σ_{in} are the transport cross sections for neutral-neutral and ion-neutral collisions, respectively). Assuming $\sigma_{in} \sim \sigma_{nn}$ [56] and $\Lambda_i \sim 1$ we find that $F_{\mathrm{T}i}$ dominates over $F_{\mathrm{T}n}$ for $n_i/n_n \gtrsim (z\tau)^{-2} \sim 10^{-4}$. For a more typical (in laboratory gas discharges) ionization fraction $n_i/n_n \sim 10^{-6}$, the ion thermal force is only slightly reducing the effect of thermophoretic force.

The electron thermal force is expected to play more important role. Spatially resolved probe measurements of the electron energy distribution function in various gas discharges have previously shown that significant electron temperature gradients can exist [57, 58]. For devices used in complex plasma research, the results from probe measurements [59], optical emission spectroscopy [60], as well as from numerical modeling [61, 62] all revealed gradients in T_e of the order of $\mathcal{O}$(eV/cm). The spatial distribution of T_e generally depends on discharge geometry, plasma parameters, and can be affected by the presence of grains [61]. Let us therefore assume $\nabla T_e = 1$ eV/cm and restrict ourselves to the comparison of the absolute magnitudes of the electron thermal force and other forces acting on a small grain in the bulk of a gas discharge. We take the plasma parameter set from Ref. [14] used to estimate relative importance of the electrical and ion drag forces: argon gas at a pressure $p = 10$ Pa ($n_n \simeq 2 \times 10^{15}$ cm^{-3}), $n_e = n_i = 3 \times 10^9$ cm^{-3}, $T_e = 1$ eV, $T_i = T_n = 0.03$ eV, $a = 1$ μm ($a/\lambda \simeq 0.04$), and $z \simeq 3$ (estimated from the collisionless OML theory). From Eqs. (6) and (7) we get $\Phi_e \simeq -22$. The electron thermal conductivity in a weakly ionized plasma is $\kappa_e \simeq (5n_eT_e/2m_e\nu_{en})$ with $\nu_{en} \simeq n_n\sigma_{en}v_{T_e}$ ($\sigma_{en} \sim 10^{-16}$ cm^2 for $T_e \sim 1$ eV in argon). This results in $F_{\mathrm{T}e} \simeq 2 \times 10^{-8}$ dyne. This force is more than three times larger than the force of gravity, experienced by the grain of this size (and material density 1.5 g/cm^3) in ground-based experiments. The equivalent electric field E_*, for which $F_{\mathrm{T}e} \simeq |Q|E_*$, is $E_* \simeq 5$ V/cm. Such a field would produce a significant plasma anisotropy, characterized by superthermal ion flows (Fig. 4b from Ref. [14]). Finally, this magnitude is comparable to the *maximum* value of the ion drag force the grain can experience in subsonic ion flows for this set of parameters ($\sim 5 \times 10^{-8}$ dyne according to Fig. 4a from Ref. [14]).

Observation of large grains, trapped in standing striations of a stratified dc glow discharge [63], provides another example where the electron thermal force can play a significant role. The electron temperature is known to increase considerably in the head of the striation, which can explain that even very massive grains can be confined there [63]. Other situations wherein plasma thermal forces can play significant role include charged aerosols, dust in planetary (e.g., Earth) atmospheres and in fusion devices. In the latter case, however, the regime of fully ionized magnetized plasma, considered recently [64], is apparently more relevant.

Altogether, there is a certain confidence that the thermal forces (especially from electrons) can have significant impact on the particle transport in many realistic situations. This finding should be properly addressed when developing new (and updating existing) numerical codes to model particle transport in various plasma environments. Expressions derived in this Letter can serve as a theoretical basis for further detailed studies.

[1] V. E. Fortov and G. E. Morfill, *Complex and dusty plasmas: From Laboratory to Space*, (CRC Press 2010).

[2] V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, Phys. Rep. **421**, 1 (2005).

[3] S. A. Khrapak, A. V. Ivlev, and G. E. Morfill, Phys. Rev. E **70**, 056405 (2004).

[4] G. E. Morfill, H. M. Thomas, U. Konopka, H. Rothermel, M. Zuzic, A. Ivlev, J. Goree, Phys. Rev. Lett. **83**, 1598 (1999).

[5] J. Goree, G. E. Morfill, V. N. Tsytovich, S. V. Vladimirov, Phys. Rev. E **59**, 7055 (1999).

[6] M. Kretschmer *et al.*, Phys. Rev. E **71**, 056401 (2005).

[7] A. M. Lipaev *et al.*, Phys. Rev. Lett. **98**, 265006 (2007).
[8] P. S. Epstein, Phys. Rev. **23**, 710 (1924).
[9] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and H. M. Thomas, Phys. Rev. E **66**, 046414 (2002).
[10] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and S. K. Zhdanov, Phys. Rev. Lett. **90**, 225002 (2003).
[11] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, S. K. Zhdanov, and H. M. Thomas, IEEE Trans. Plasma Sci. **32**, 555 (2004).
[12] A. V. Ivlev, S. A. Khrapak, S. K. Zhdanov, G. E. Morfill, and G. Joyce, Phys. Rev. Lett. **92**, 205007 (2004).
[13] A. V. Ivlev, S. K. Zhdanov, S. A. Khrapak, and G. E. Morfill, Phys. Rev. E **71**, 016405 (2005).
[14] S. A. Khrapak, A. V. Ivlev, S. K. Zhdanov, and G. E. Morfill, Phys. Plasmas **12**, 042308 (2005).
[15] I. H. Hutchinson, Plasma Phys. Control. Fusion **47**, 71 (2005); Plasma Phys. Control. Fusion **48**, 185 (2006).
[16] L. Patacchini and I. H. Hutchinson, Phys. Rev. Lett. **101**, 025001 (2008).
[17] V. Ikkurthi, K. Matyash, A. Melzer, and R. Schneider, Phys. Plasmas **16**, 043703 (2009).
[18] C. Zafiu, A. Melzer, and A. Piel, Phys. Plasmas **9**, 4794 (2002); S. A. Khrapak *et al.*, Phys. Plasmas **10**, 4579 (2003).
[19] M. Hirt, D. Block. and A. Piel, IEEE Trans. Plasma Sci. **32**, 582 (2004); Phys. Plasmas **11**, 5690 (2004).
[20] V. Yaroshenko *et al.*, Phys. Plasmas **12**, 093503 (2005).
[21] M. Wolter, A. Melzer, O. Arp, M. Klindworth, and A. Piel, Phys. Plasmas **14**, 123707 (2007).
[22] V. Nosenko, R. Fisher, R. Merlino, S. Khrapak, G. Morfill, and K. Avinash, Phys. Plasmas **14**, 103702 (2007); S. A. Khrapak, V. Nosenko, G. E. Morfill, and R. Merlino, Phys. Plasmas **16**, 044507 (2009).
[23] C. M. Ticos *et al.*, Phys. Rev. Lett. **100**, 155002 (2008).
[24] M. H. Thoma, J. Phys. G: Nucl. Part. Phys. **31**, L7 (2005); S. Mrówczyński and M. H. Thoma, Annu. Rev. Nucl. Part. Sci. **57**, 61 (2007).
[25] J. L. Feng, M. Kaplinghat, and H. B. Yu, Phys. Rev. Lett. **104**, 151301 (2010); A. Loeb and N. Weiner, Phys. Rev. Lett. **106**, 171302 (2011).
[26] S. D. Baalrud, Phys. Plasmas **19**, 030701 (2012).
[27] S. A. Khrapak and G. E. Morfill, Phys. Rev. E **69**, 066411 (2004).
[28] J. R. Brook, J. Colloid Interface Sci. **25**, 564 (1967).
[29] A. Einstein, Z. Phys. **27**, 1 (1924).
[30] L. Waldmann, Z. Naturforsch. **14a**, 589 (1959).
[31] L. Talbot, R. K. Cheng, R. W. Schefer, and D. R. Willis, J. Fluid Mech. **101**, 737 (1980).
[32] Z. Li and H. Wang, Phys. Rev. E **70**, 021205 (2004).
[33] G. M. Jellum, J. E. Daugherty, and D. B. Graves, J. Appl. Phys. **69**, 6923 (1991).
[34] H. Rothermel, T. Hagl, G. E. Morfill, M. H. Thoma, and H. M. Thomas, Phys. Rev. Lett. **89**, 175001 (2002).
[35] M. Schwabe, M. Rubin-Zuzic, S. Zhdanov, H. M. Thomas, and G. E. Morfill. Phys. Rev. Lett. **99**, 095002 (2007).
[36] M. Schwabe, M. Rubin-Zuzic, S. Zhdanov, A. V. Ivlev, H. M. Thomas, and G. E. Morfill, Phys. Rev. Lett. **102**, 255005 (2009).
[37] C. Schmidt, O. Arp, and A. Piel, Phys. Plasmas **18**, 013704 (2011).
[38] V. A. Schweigert, Plasma Phys. Reports **27**, 997 (2001).
[39] D. Else, R. Kompaneets, and S. V. Vladimirov, Phys. Plasmas **16**, 062106 (2009).
[40] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases* (Cambridge University Press, Cambridge, 1970).
[41] S. Khrapak and G. Morfill, Contrib. Plasma Phys. **49**, 148 (2009).
[42] J. E. Daugherty, R. K. Porteous, M. D. Kilgore, and D. B. Graves, J. Appl. Phys. **72**, 3934 (1992).
[43] S. Ratynskaia, U. de Angelis, S. Khrapak, B. Klumov, and G. E. Morfill, Phys. Plasmas **13**, 104508 (2006).
[44] V. N. Tsytovich, Phys. Usp. **167**, 57 (1997).
[45] J. E. Allen, B. M. Annaratone, U. de Angelis, J. Plasma Phys. **63**, 299 (2000).
[46] A. V. Filippov, A. G. Zagorodny, A. F. Pal', A. N. Starostin, and A. I. Momot, JETP Lett. **86**, 761 (2007).
[47] S. A. Khrapak, B. A. Klumov, and G. E. Morfill, Phys. Rev. Lett. **100**, 225003 (2008).
[48] S. A. Khrapak, A. V. Ivlev, and G. E. Morfill, Phys. Plasmas **17**, 042107 (2010).
[49] J. E. Allen, Phys. Scr. **45**, 497 (1992).
[50] In a recent experimental study of particle charging in a dc glow discharge at high pressures [S. A. Khrapak *et al.*, EPL **97**, 35001 (2012)] the values of $z \sim 0.3$ and $a/\lambda \sim 0.02$ have been reported, which correspond to the regime $\Phi_e > 0$.
[51] S. A. Khrapak, S. K. Zhdanov, A. V. Ivlev, and G. E. Morfill, J. Appl. Phys. **101**, 033307 (2007).
[52] S. V. Vladimirov, S. A. Khrapak, M. Chaudhuri, and G. E. Morfill, Phys. Rev. Lett. **100**, 055002 (2008).
[53] A. V. Filippov, A. G. Zagorodny, and A. I. Momot, JETP Lett. **88**, 24 (2008).
[54] S. A. Khrapak, M. Chaudhuri, and G. E. Morfill, IEEE Trans. Plasma Sci. **37**, 487 (2009).
[55] The potential barrier emerges for $\beta(v) \gtrsim 13.2$ [10].
[56] For ions in their parent gases σ_{in} is normally several times larger than σ_{nn} due to resonance charge exchange.
[57] V. A. Godyak and R. B. Piejak, Appl. Phys. Lett. **63**, 3137 (1993).
[58] S. P. Fusselman, H. K. Yasuda, J. B. Javedani, J. Chiang, and M. A. Prelas, J. Vac. Sci. Technol A **12**, 3115 (1994).
[59] M. Wolter, A. Melzer, O. Arp, M. Klindworth, and A. Piel, Phys. Plasmas **14**, 123707 (2007).
[60] S. Mitic, B. A. Klumov, M. Y. Pustilnik, and G. E. Morfill, JETP Lett. **91**, 231 (2010).
[61] V. Land and W. J. Goedheer, New. J. Phys. **9**, 246 (2007).
[62] O. Arp, J. Goree, and A. Piel, Phys. Rev. E **85**, 046409 (2012).
[63] A. M. Lipaev *et al.*, JETP **85**, 1110 (1997).
[64] A. A. Stepanenko, R. D. Smirnov, V. M. Zhdanov, and S. I. Krasheninnikov, Phys. Plasmas **18**, 033702 (2011).